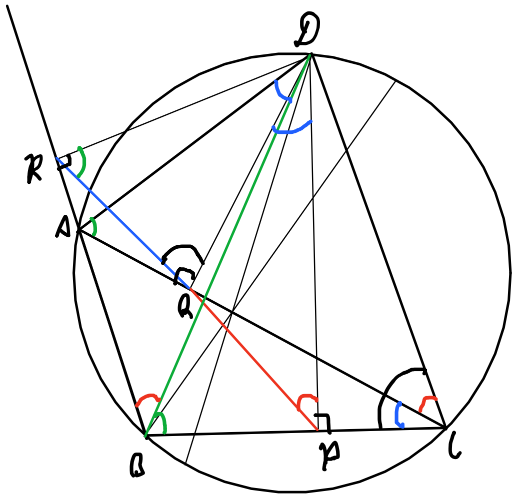
Enoch Yu

Let $ABCD$ be a cyclic quadrilateral. Let P, Q, and R be the feet of perpendiculars from D to lines $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Show that $PQ = QR$ if and only if the bisectors of angles ABC and ADC meet on segment $\overline{AC}$.



i) If the angle bisectors meet at AC say E,
$PQ = QR$.

$AD : DC = AE : EC$

$AB : BC = AE : EC$

$AE : EC = AF : FC$

$E \equiv F$

$\therefore AD : DC = AB : BC$

$AD : AB = DC : BC$

$DQAR$, $DCPQ$, $DPBR \Rightarrow$ cyclic

$\triangle ABD \sim \triangle QPD$ (AA)

$\triangle DBC \sim \triangle DRQ$ (AA)

$AB : AD = PQ : QD$

$BC : DC = RQ : QD$

$PQ : \cancel{QD} = RQ : \cancel{QD}$

$\therefore RQ = PQ$

Reversible.

$\square$