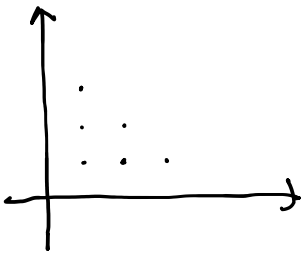
Enoch Yu

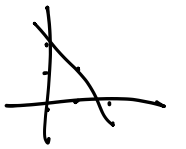
A line in the plane is called sunny if it is not parallel to any of the x–axis, the y–axis, and the line $x+y=0$.

Let $n \geq 3$ be a given integer. Determine all nonnegative integers k such that there exist n distinct lines in the plane satisfying both of the following:

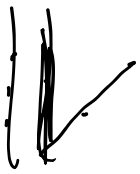
- for all positive integers a and b with $a+b \leq n+1$, the point (a,b) is on at least one of the lines; and
- exactly k of the n lines are sunny.

$n=3 \Rightarrow a+b \leq 4$

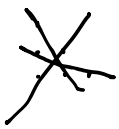




$\Rightarrow k=0$



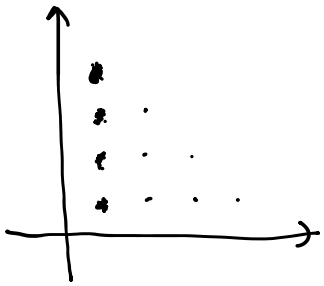
$\Rightarrow k=1$



$\Rightarrow k=3$

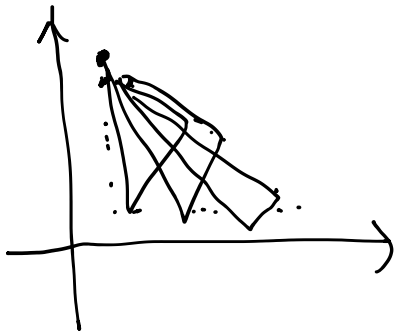
$k \in \{0,1,3\}$

$n=4$



$$\frac{\frac{(n+1)n}{2}}{n} = \frac{n+1}{2}$$

$n=n$



"$n-1$"

"$n-2$"

$\vdots$

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