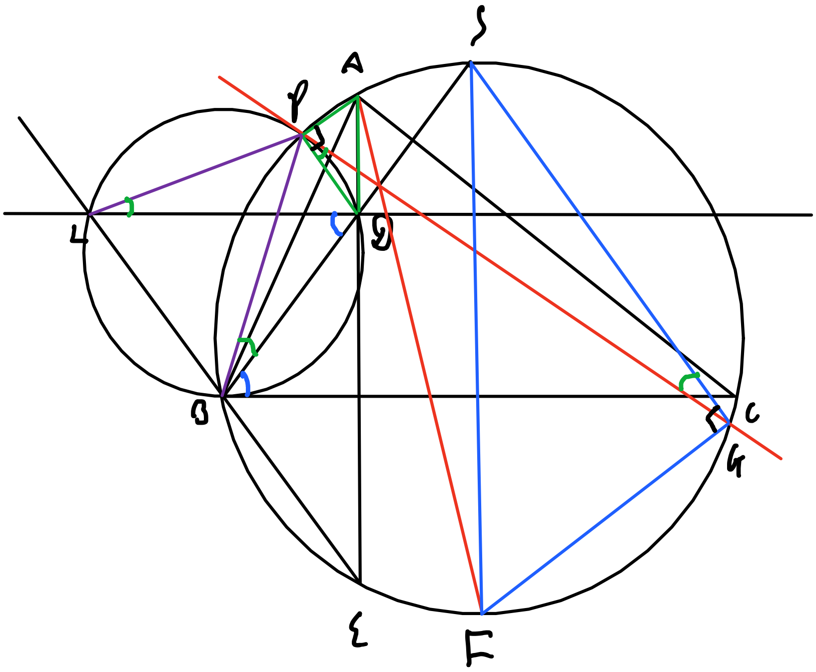
Enoch Yu

Let ABC be an acute-angled triangle with $AB < AC$. Let Ω be the circumcircle of ABC. Let S be the midpoint of the arc CB of Ω containing A. The perpendicular from A to BC meets BS at D and meets Ω again at $E \neq A$. The line through D parallel to BC meets line BE at L. Denote the circumcircle of triangle BDL by ω. Let ω meet Ω again at $P \neq B$. Prove that the line tangent to ω at P meets line BS on the internal angle bisector of $\angle BAC$.



- $AE \parallel SF$
- $SF \Rightarrow$ diameter
- $H := AF \cap BS$
- $\triangle AHD \sim \triangle FHS$
- $\angle PAD = 180 - \angle PBE$

  $= \angle PBL = \angle PDL$

$\angle PDL + \angle APD = 90^\circ$

$\angle PAD + \angle APD = 90^\circ$

- $PD \parallel SG$

$\therefore \triangle APD \sim \triangle FGS$

$\therefore H \Rightarrow$ center of homothety

$\Rightarrow$ center of perspectivity

$\square$

- $I := AF \cap PG$

$AD : SF = AH : HF$

$= AP : GF = AI : IF$

$\therefore H \equiv I$ $\quad$ $AF \cap PG = AF \cap BS$

$\square$