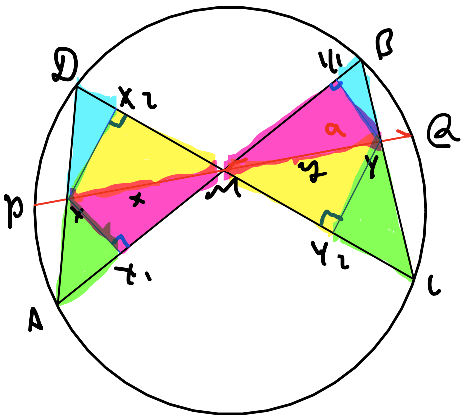
Enoch Yu

The chords AB and CD of a circle intersect at M, which is the midpoint of the chord PQ. The points X and Y are the intersections of the segments AD and PQ, respectively, and BC and PQ, respectively. Show that M is the midpoint of XY.



pf 1)

$x : y = XX_2 : YY_2 = \cancel{X_2 M : MY_2}$

$x : y = XX_1 : YY_1 = \cancel{X_1 M : MY_1}$

$XX_2 : YY_1 = DX : BQ = \cancel{DX_2 : BY_1}$

$XX_1 : YY_2 = AX : CQ = \cancel{AX_1 : CY_2}$

$$\frac{x}{y} = \frac{XX_2}{YY_2} = \frac{XX_1}{YY_1} \qquad \frac{XX_2}{YY_1} = \frac{DX}{BQ} \qquad \frac{XX_1}{YY_2} = \frac{AX}{CQ}$$

$$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$$

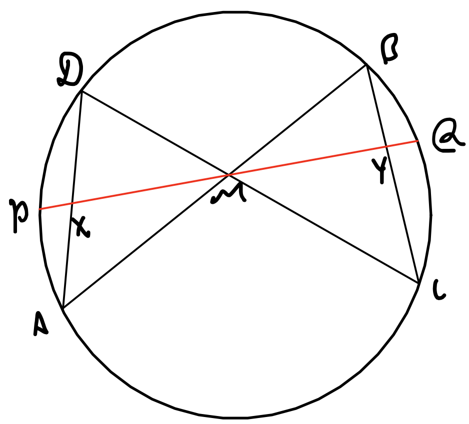
$$\frac{DX}{BQ} \cdot \frac{AX}{CQ} = \frac{XX_2}{YY_1} \cdot \frac{XX_1}{YY_2} = \frac{XX_1}{YY_1} \cdot \frac{XX_2}{YY_2} = \frac{x^2}{y^2}$$

$$\frac{PX \cdot XQ}{PY \cdot YQ} = \frac{x^2}{y^2} = \frac{(a-x)(x+a)}{(a+y)(a-y)} = \frac{a^2 - x^2}{a^2 - y^2} = \frac{a^2 - x^2 + x^2}{a^2 - y^2 + y^2} = 1$$

$$x = y$$

$\square$

pf 2)



$$(P, C; A, Q) = (PD, DC; DA, DQ)$$
$$= (P, M; X, Q)$$
$$= \frac{PX}{MX} \Big/ \frac{PQ}{MQ} = \frac{PX \cdot MQ}{MX \cdot PQ}$$
$$= (PB, BC; BA, BQ)$$
$$= (P, Y; M, Q)$$
$$= \frac{PM}{\cdot} \Big/ \frac{PQ}{} = \frac{PM \cdot YQ}{\cdot}$$

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$YM \,/\, YQ = YM \cdot PB$

$$\frac{PX \cdot \cancel{MB}}{MX \cdot \cancel{PB}} = \frac{\cancel{PM} \cdot YB}{YM \cdot \cancel{PB}}$$

$$\frac{PX}{MX} = \frac{YB}{YM} = \frac{a-x}{x} = \frac{a-y}{y}$$

$$ax - \cancel{xy} = ay - \cancel{xy}$$

$$x = y$$

D