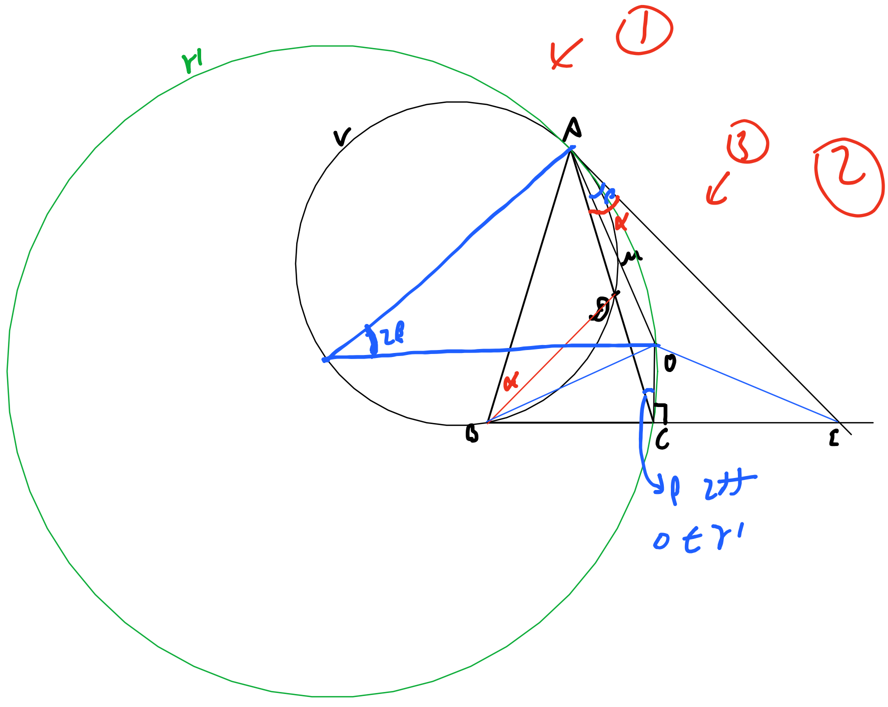
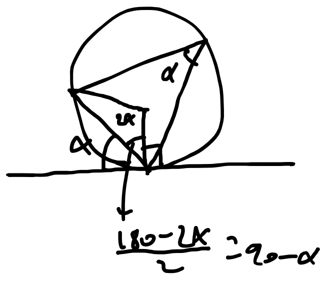
Enoch Yu

Let ABC be an acute triangle with $AB = AC$, let D be the midpoint of the side AC, and let γ be the circumcircle of the triangle ABD. The tangent of γ at A crosses the line BC at E. Let O be the circumcenter of the triangle ABE. Prove that midpoint of the segment AO lies on γ.



$O \in \gamma'$?



- $\angle DCB = \angle ABC$
- $\angle BDC = \angle BAC + \alpha$
  $= \angle BAE$
- $\therefore \triangle DCB \sim \triangle ABE$
- $\because 2DC = AC = AB$
- $\therefore 2BC = BE$

- $\angle ACO = 90 - \angle ACB = 90 - \angle ABE$
  $= (\angle ABO + \angle OBE + \angle OAE) - \angle ABE$
  $= \cancel{\angle ABE} + \angle OAE - \cancel{\angle ABE}$
  $= \angle OAE$

$\square$