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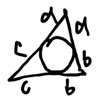
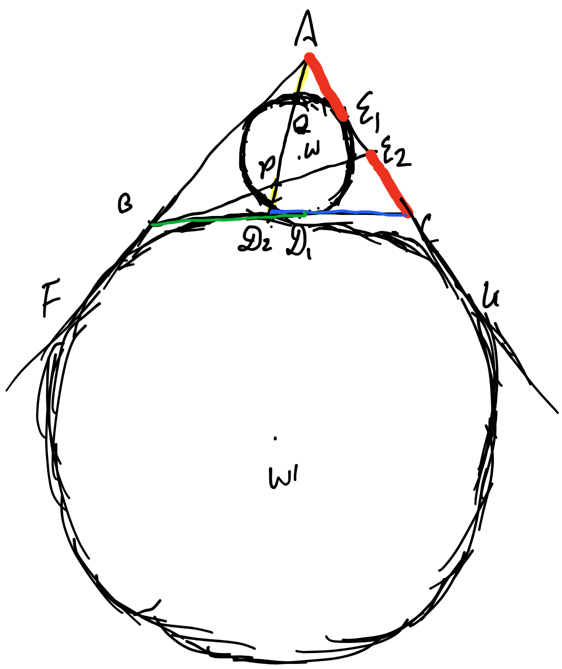
Let ABC be a triangle and let ω be its incircle. Denote by D_1 and E_1 the points where ω is tangent to sides BC and AC, respectively. Denote by D_2 and E_2 the points on sides BC and AC, respectively, such that $CD_2 = BD_1$ and $CE_2 = AE_1$, and denote by P the point of intersection of segments AD_2 and BE_2. Circle ω intersects segment AD_2 at two points, the closer of which to the vertex A is denoted by Q. Prove that $AQ = D_2P$.



$a = s-(b+c)$



1

- ω' is tangent to BC at D_2.

2

- ω and ω' are homothetic with respect to A.

3

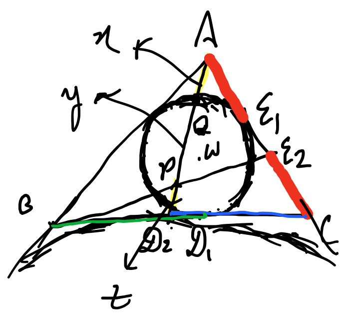
$\therefore AQ : QD_2 = AE_1 : E_1G$

Let $AB = c$, $BC = a$, $CA = b$, and $s = \dfrac{a+b+c}{2}$.

$$E_1G = E_1C + CG = E_1C + CD_2 = CD_1 + CD_2 = CD_1 + BD_1 = a$$

$$\frac{AQ}{QD_2} = \frac{AE_1}{E_1G} = \frac{s-a}{a}$$

4



$\triangle ABC$, BE_2

$$\frac{CE_2}{E_2A} \cdot \frac{AP}{PD_2} \cdot \frac{D_2B}{BC} = 1$$

$$\frac{AP}{PD_2} = \frac{E_2A \cdot BC}{CE_2 \cdot D_2B} = \frac{CE_1 \cdot a}{(s-a)\, CD_1} = \frac{\cancel{(b-(s-a))}\,a}{(s-a)\cancel{(a-(s-b))}}$$

$$= \frac{a}{s-a}$$

$$\frac{AQ}{QD_2} = \frac{PD_2}{AP}$$

$$\frac{x}{y+z} = \frac{z}{x+y}$$

$$x^2 + xy = yz + z^2$$

$$(x-z)(x+z) = y(z-x)$$

i) $\underline{x = z}$

$0 = 0$

ii) $x \neq z$

$x+z=-y$

$x+z+y=0$

$\therefore \boxed{\Delta \alpha = \rho \alpha_2}$