Enoch Yu

Professor Oak is feeding his 100 Pokémon. Each Pokémon has a bowl whose capacity is a positive real number of kilograms. These capacities are known to Professor Oak. The total capacity of all the bowls is 100 kilograms. Professor Oak distributes 100 kilograms of food in such a way that each Pokémon receives a non-negative integer number of kilograms of food (which may be larger than the capacity of their bowl). The *dissatisfaction* level of a Pokémon who received N kilograms of food and whose bowl has a capacity of C kilograms is equal to $|N - C|$.

Find the smallest real number D such that, regardless of the capacities of the bowls, Professor Oak can distribute the food in a way that the sum of the dissatisfaction levels over all the 100 Pokémon is at most D.

O 100kg

C_1 C_2 C_3 ... C_{98} C_{99} C_{100} $\in \mathbb{R}^+$ $\quad + \Rightarrow 100 \qquad 49.5$

N_1 N_2 N_3 ... N_{98} N_{99} N_{100} $\in \mathbb{N}_0$

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i) $C_1 = C_2 = \cdots = C_{99} = 0.5$, $C_{100} = 100 - 0.5 \cdot 99 = 50.5$

$N_1 = N_2 = \cdots = N_{50} = 1$, $N_{51} = \cdots = N_{99} = 0$, $N_{100} = 50$

$0.5 \cdot 50 + 0.5 \cdot 49 + 0.5 = 0.5 \cdot 100 = 50$

$50 \le D$

ii) $C_i = I_i + F_i$

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$I_1 + I_2 + \cdots + I_{100} \le 100$

$I_1+F_1 \quad I_2+F_2 \quad I_3+F_3 \quad \cdots \quad I_{98}+F_{98} \quad I_{99}+F_{99} \quad I_{100}+F_{100}$

$F_1 + F_2 + F_3 + \cdots + F_{98} + F_{99} + F_{100} \in \mathbb{N}_0$

$k := 100 - \sum_{i=1}^{100} I_i = \sum_{i=1}^{100} F_i$

dissatisfaction level $= |I_1 - I_1 - F_1| + |I_2 - I_2 - F_2| + \cdots + |I_{100} + 1 - I_{100} - F_{100}|$

$= |I_1 - I_1 - F_1| + \cdots + |I_{100-k} - I_{100-k} - F_{100-k}| + |I_{100-k+1} + 1 - I_{100-k+1} - F_{100-k+1}| + \cdots + |I_{100} + 1 - I_{100} - F_{100}|$

$= F_1 + F_2 + \cdots + F_{100-k} + (1 - F_{100-k+1}) + (1 - F_{100-k+2}) + \cdots + (1 - F_{100})$

$= F_1 + \cdots + F_{\ldots} + k - (F_{\ldots} + \cdots + F_{\ldots})$

$$= F_1 + \cdots + F_{100-k} + k - (F_{100-k+1} + \cdots + F_{100})$$

$$= F_1 + \cdots + F_{100-x} + F_1 + \cdots \cancel{+ F_{100}} - \cancel{(F_{100-x+1} + \cdots + F_{100})}$$

$$= 2(F_1 + \cdots + F_{100-x})$$

WLOG, $F_1 \le F_2 \le \cdots \le F_{100}$

$$\frac{F_1 + \cdots + F_{100-x}}{100-x} \le \frac{F_1 + F_2 + \cdots + F_{100}}{100} = \frac{x}{100}$$

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$\downarrow$

$$F_1 + \cdots + F_{100-x} \le \frac{x(100-x)}{100} \le \frac{2500}{100}$$

$$x + (100-x) \ge 2\sqrt{x(100-x)}$$

$$50 \ge \sqrt{x(100-x)}$$

$$2500 \ge x(100-x)$$

$$2(F_1 + \cdots + F_{100-x}) \le 50$$

$$50 \le D \le 50$$

$$\therefore \boxed{D = 50}$$

$$d = (1-F_1) + \cdots + (1-F_x) + F_{x+1} + \cdots + F_{100}$$

$$= x - (F_1 + \cdots + F_x) + F_{x+1} + \cdots + F_{100}$$

$$= 2(F_{x+1} + \cdots + F_{100})$$

WLOG, let $F_1 \ge F_2 \ge \cdots \ge F_{100}$

$$\frac{F_{x+1} + \cdots + F_{100}}{100-x} \le \frac{F_1 + \cdots + F_{100}}{100}$$

$$F_{x+1} + \cdots + F_{100} \le \frac{x(100-x)}{100} \le 25$$

$$\therefore d \le 50$$

$$50 \le D \le 50$$

$$\therefore D = 50$$