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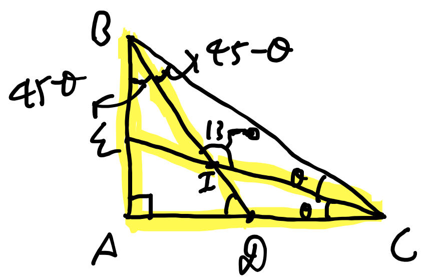
Let ABC be a triangle with $\angle A = 90°$. Points D and E lie on sides AC and AB, respectively, such that $\angle ABD = \angle DBC$ and $\angle ACE = \angle ECB$. Segments BD and CE meet at I. Determine whether or not it is possible for segments AB, AC, BI, ID, CI, IE to all have integer lengths.



I ⇒ incenter

$$\frac{90-2\theta}{2} = 45-\theta$$

Assume that $AB, AC, BI, IC \in \mathbb{Z}^+$.

$$\cos 135° = \frac{BI^2 + IC^2 - BC^2}{2 \cdot BI \cdot IC}$$

$$-\frac{\sqrt{2}}{2} = \frac{BI^2 + IC^2 - AB^2 - AC^2}{2 \cdot BI \cdot IC}$$

By contradiction, $AB, AC, BI, IC \in \mathbb{Z}^+$ is false.