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Let $a_0 > 0$ be a real number, and let

$$a_n = \frac{a_{n-1}}{\sqrt{1 + 2020 \cdot a_{n-1}^2}}, \quad \text{for } n = 1, 2, \ldots, 2020.$$

①

Show that $a_{2020} < \frac{1}{2020}$. ②

$$\frac{1}{a_n^2} = \frac{1 + 2020 \cdot a_{n-1}^2}{a_{n-1}^2} = \frac{1}{a_{n-1}^2} + 2020$$ ③

$$\therefore \frac{1}{a_n^2} - \frac{1}{a_{n-1}^2} = 2020$$

④

$$\frac{1}{a_{2020}^2} - \cancel{\frac{1}{a_{2019}^2}} + \cancel{\frac{1}{a_{2019}^2}} - \cancel{\frac{1}{a_{2018}^2}} \cancel{+} \cancel{\cdots} \cancel{+} \cancel{\frac{1}{a_1^2}} - \frac{1}{a_0^2} = 2020 \cdot 2020$$

$$\frac{1}{a_{2020}^2} - \frac{1}{a_0^2} = 2020^2$$

$$\frac{1}{a_{2020}^2} > \frac{1}{a_{2020}^2} - \frac{1}{a_0^2} = 2020^2$$

By Induction, $a_{2020} > 0$.

$$\frac{1}{a_{2020}^2} > 2020^2$$

$$a_{2020}^2 < \frac{1}{2020^2}$$

$$\therefore a_{2020} < \frac{1}{2020}$$

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