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Find a continuous function $f: \mathbb{R} \to \mathbb{R}$ s.t. $f(x+y) = f(x)f(y)$.

(solution)

If $x = y = 0$,

$f(0) = f(0)^2$

$\therefore f(0) = 0$ or $f(0) = 1$

i) $f(0) = 0$

If $y = 0$

$f(x) = f(x)f(0) = 0$

$\therefore \boxed{f(x) = 0}$

ii) $f(0) = 1$

$x = y \Rightarrow f(2x) = f(x)^2$

$y = 2x \Rightarrow f(3x) = f(x)^3$

$y = 3x \Rightarrow f(4x) = f(x)^4$

$\therefore$ By induction

$f(nx) = f(x)^n$

If $x = \frac{1}{m}$ $(m \neq 0)$

$f(\frac{n}{m}) = f(\frac{1}{m})^n$

If $n = m$, $x = \frac{1}{m}$

$f(1) = f(\frac{1}{m})^m = C$

$f(\frac{1}{m})^n = f(\frac{1}{m})^{m \cdot \frac{n}{m}} = C^{\frac{n}{m}}$

$f(\frac{n}{m}) = C^{\frac{n}{m}}$

$\therefore \boxed{f(x) = C^x}$

let $f(1) = C$

$C > 0$

$$\boxed{f(x) = e^{Cx}}$$