

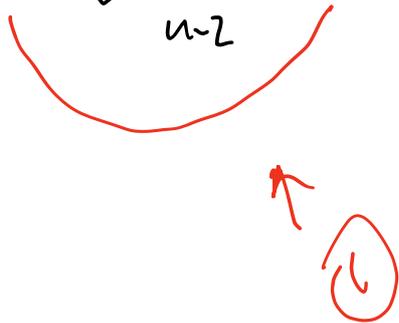
For $n \geq 2$ let a_1, a_2, \dots, a_n be positive real numbers such that

$$(a_1 + a_2 + \dots + a_n) \left(\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \right) \leq \left(n + \frac{1}{2} \right)^2.$$

Prove that $\max(a_1, a_2, \dots, a_n) \leq 4 \min(a_1, a_2, \dots, a_n)$.

WLOG, let $a_1 \geq a_2 \geq \dots \geq a_n$

$$(a_n + a_2 + \dots + a_{n-1} + a_1) \left(\frac{1}{a_1} + \dots + \frac{1}{a_n} \right) \geq \left(\sqrt{\frac{a_n}{a_1}} + n-2 + \sqrt{\frac{a_1}{a_n}} \right)^2$$



$$\left(n + \frac{1}{2} \right)^2 \geq \left(\sqrt{\frac{a_n}{a_1}} + \sqrt{\frac{a_1}{a_n}} + n-2 \right)^2$$

$$n + \frac{1}{2} \geq n-2 + \sqrt{\frac{a_n}{a_1}} + \sqrt{\frac{a_1}{a_n}}$$

$$\frac{27}{4} \geq \frac{a_n}{a_1} + \frac{a_1}{a_n} + 2$$

$$\frac{17}{4} \geq \frac{a_n}{a_1} + \frac{a_1}{a_n}$$

$$\frac{17}{4} a_1 a_n \geq a_n^2 + a_1^2$$

$$4a_1 - a_n > 0$$

$$4a_1^2 + a_n^2 - 17a_1 a_n \leq 0$$

$$(4a_1 - a_n)(a_1 - 4a_n) \leq 0$$

$$a_1 \leq 4a_n$$

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