Enoch Yu

Let a, b, c be positive real numbers such that $a+b+c = 4\sqrt[3]{abc}$. Prove that

$$2(ab+bc+ca) + 4\min(a^2, b^2, c^2) \geq a^2+b^2+c^2.$$

WLOG, let $a \geq b \geq c$.

$$2(ab+bc+ca) + 4c^2 \geq a^2+b^2+c^2$$

$$4(ab+bc+ca) + 4c^2 \geq (a+b+c)^2$$

$$4c(a+b+c) + 4ab \geq 16(abc)^{\boxed{\frac{2}{3}}}$$

$$\boxed{c(4\sqrt[3]{abc}) + ab} \geq \underline{4(abc)^{\frac{2}{3}}}$$

Don't be Scared!!

$$c(4\sqrt[3]{abc}) + ab \geq 2\sqrt{4abc\sqrt[3]{abc}}$$

$$= 4\left((abc)^{\frac{4}{3}}\right)^{\frac{1}{2}}$$

$$= \underline{4(abc)^{\frac{2}{3}}}$$

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