Enoch Yu

Let a, b, c be positive reals such that $abc = 1$. Show that

$$\frac{1}{a^5(b+2c)^2}+\frac{1}{b^5(c+2a)^2}+\frac{1}{c^5(a+2b)^2}\geq\frac{1}{3}.$$

✓ 1

$$\left(\sum_{cyc}\frac{1}{a^5(b+2c)^2}\right)^{\frac{1}{3}}\left(\sum_{cyc}a(b+2c)\right)^{\frac{1}{3}}\left(\sum_{cyc}a(b+2c)\right)^{\frac{1}{3}}\geq\sum_{cyc}\frac{1}{a}$$

$$\left(\sum_{cyc}\frac{1}{a^5(b+2c)^2}\right)\left(\sum_{cyc}a(b+2c)\right)^2\geq\left(\sum_{cyc}\frac{1}{a}\right)^3$$

$$=(ab+bc+ca)^3$$

$$\geq 3(ab+bc+ca)^2$$

$$\left(\sum_{cyc}a(b+2c)\right)^2=(ab+2ca+bc+2ab+ca+2bc)^2$$

$$=(3ab+3bc+3ca)^2$$

$$=9(ab+bc+ca)^2$$

$$\therefore\sum_{cyc}\frac{1}{a^5(b+2c)^2}\geq\frac{1}{3}$$

$\square$