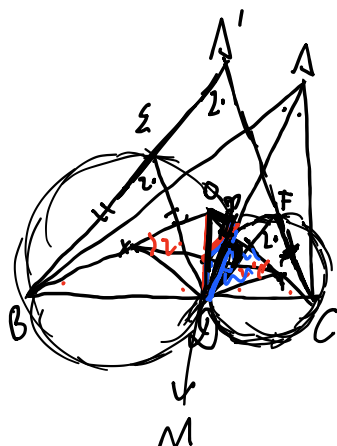For an acute scalene triangle ABC, let D be the intersection of the side BC and the angle bisector of angle A. Moreover, let E and F be the circumcenter of $\triangle ABD$ and $\triangle ADC$ respectively. Assuming that $P(\neq D)$ is the point of intersection of the circumcircle of $\triangle BDE$ and $\triangle DCF$ and O, X, Y are the circumcenter of $\triangle ABC$, $\triangle BDE$, and $\triangle DCF$ respectively, show that $OP \parallel XY$.

(1) Euler line

(2) Length Bound

(3) Radical Axis

(4) Own property

$\triangle A'BC$ is cyclic.

$\therefore \angle BA'C = \angle BAC$.

$OX D Y$ is a parallelogram.

$\therefore OM = MD = MP$

$\implies \angle OPD = 90°$

$PD \perp XY$ because

PD is the radical

axis of X and Y.

$\therefore OP \parallel XY$

$\square$