

Compute $(19! + 19, 20! + 19)$.

Wilson's Theorem

$\forall p \in \mathbb{N}$, p is a prime iff $p \mid (p-1)! + 1$

$(a, b) = (a, an \pm b)$ ①

let $d \mid a, d \mid b \Rightarrow d \mid an \pm b \Rightarrow (n, an \pm b)$ subsets of (a, b)
 let $d \mid a$ (4.5)
 $d \mid an \pm b$ (4.5)
 $\therefore d \mid b$ (4.5)

Because 19 is a prime, $19 \mid 18! + 1$.

$$\begin{aligned} (19! + 19, 20! + 19) &= (19! + 19, 20! + 20 \cdot 19 - 20! - 19) \\ &= (19! + 19, 19 \cdot 19) \quad \text{②} \\ &= (19(18! + 1), 19 \cdot 19) \end{aligned}$$

let $d = (19! + 19, 20! + 19)$.

$19 \mid d,$
 $361 \mid d.$

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