Enoch Yu

Solve for (x, y) in each equation.

$x^2 - 11y^2 = 1 \Rightarrow \boxed{(x_n, y_n) = (10+3\sqrt{11})^n}$

$x^2 - 11y^2 = -1$

$\hookrightarrow$ $\boxed{\text{no integer solutions}}$

$\sqrt{D} \Rightarrow$ odd periods

$D = 11$

$x^2 - 11y^2 = 1$

Fundamental solution

$\sqrt{11} = 3 + \sqrt{11} - 3 \quad = \langle 3; \overline{3\,6} \rangle$ 2

$= 3 + \cfrac{1}{\cfrac{1}{\sqrt{11}-3}}$

$3 = \frac{3}{1}$ ×

$3 + \frac{1}{3} = \frac{10}{3}$

$= 3 + \cfrac{1}{\cfrac{\sqrt{11}+3}{2}}$

$= 3 + \cfrac{1}{3 + \cfrac{\sqrt{11}-3}{2}}$

$= 3 + \cfrac{1}{3 + \cfrac{1}{\cfrac{2}{\sqrt{11}-3}}}$

$= 3 + \cfrac{1}{3 + \cfrac{1}{3+\sqrt{11}}}$

$= 3 + \cfrac{1}{3 + \cfrac{1}{3+3+\cfrac{1}{3+\cfrac{1}{3+\sqrt{11}}}}}$

$= 3 + \cfrac{1}{3 + \cfrac{1}{6+\cfrac{1}{3+\cfrac{1}{3+\sqrt{11}}}}}$

$(x_1, y_1) = (10, 3)$

$(x_n, y_n) = (x_1 + y_1\sqrt{d})^n$

$(x_2, y_2) = (10+3\sqrt{11})^2 \Rightarrow 100 + 99 + 60\sqrt{11}$

$= (199, 60)$

$(200-1)^2 - 11 \cdot 60^2 = \cancel{40000} - \cancel{400} + 1 - \cancel{39600}$

$= 1$ //