Enoch Yu

Pascal's Identity

$0 < k \le n$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

pf 1) Assume there are $n+1$ students. We are going to form a committee of k students, which is $\binom{n+1}{k}$. We have two possible cases: 1. Bob is present 2. Bob is not in the committee.

1. Bob is present $\Rightarrow \binom{n}{k-1}$
2. Bob is not present $\Rightarrow \binom{n}{k}$

$+ \quad = \binom{n+1}{k}$

$\square$

pf 2)

$$\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!} = \frac{n!(n-k+1) + n!\cdot k}{k!(n-k+1)!}$$

$$= \frac{n!(n+1)}{k!(n-k+1)!}$$

$$= \frac{(n+1)!}{k!(n-k+1)!}$$

$$= \binom{n+1}{k}$$

$\square$

pf 3)

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

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$$\vdots$$

$$\binom{n}{k-1} \quad \binom{n}{k} \quad \binom{n}{k+1}$$

$$\binom{n+1}{k} \quad \binom{n+1}{k+1}$$

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

$\square$