

Pascal's Identity

$$0 < k \leq n$$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

PF1) Assume there are  $n+1$  students. We are going to form a committee of  $k$  students, which is  $\binom{n+1}{k}$ . We have two possible cases: 1. Bob is present 2. Bob is not in the committee.

1. Bob is present  $\Rightarrow \binom{n}{k-1}$

2. Bob is not present  $\Rightarrow \binom{n}{k}$

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k} \quad \square$$

PF2)

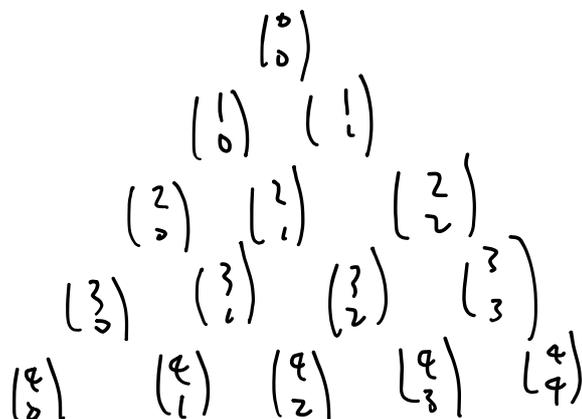
$$\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!} = \frac{n!(n-k+1) + n! \cdot k}{k!(n-k+1)!}$$

$$= \frac{n!(n+1)}{k!(n-k+1)!}$$

$$= \frac{(n+1)!}{k!(n-k+1)!}$$

$$= \binom{n+1}{k} \quad \square$$

PF3)



$$\begin{array}{ccc} & & \vdots \\ & & \vdots \\ & \binom{n}{k-1} & \binom{n}{k} & \binom{n}{k+1} \\ & \binom{n+1}{k} & \binom{n+1}{k+1} & \end{array}$$

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

D