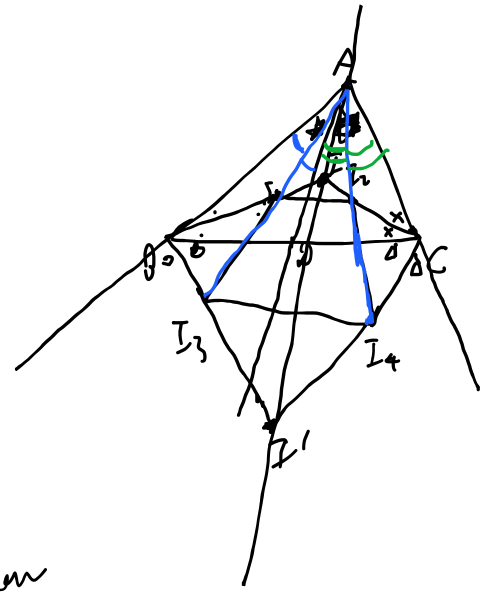
Enoch Yu

D is a point on $\overline{BC}$ in $\triangle ABC$. Let I_1, I_2 be the incenter of $\triangle ABD$ and $\triangle ACD$ respectively. Moreover, let I_3 and I_4 be ex-centers in respect to $\angle BAD$ and $\angle CAD$ respectively. Show that $\overline{I_1I_2}$, $\overline{I_3I_4}$, and $\overline{BC}$ concur.



I is the incenter of $\triangle ABC$ since BI and IC are angle bisectors of angle B and C respectively.

Similarly I' is the excenter of $\triangle ABC$ since BI' and CI' are angle bisectors of angle B and C respectively.

I: $BI_1 \cap CI_2$
A: $I_1I_3 \cap I_2I_4$
I': $BI_3 \cap CI_4$

$\Big\}$ collinear

AI and AI' are both angle bisectors of angle A. Thus, A, I, I' are collinear. Moreover, because AI_1 and AI_3 are angle bisectors of $\angle BAD$, I_3, I_1, A are collinear. Similarly, I_4, I_2, and A are collinear. Thus, by Desargues theorem, $\overline{I_1I_2}$, $\overline{I_3I_4}$, BC concur.