Enoch Yu

Solve for x modulo 30 that satisfies the following congruences:

$$x \equiv 3 \pmod{2}$$
$$x \equiv 5 \pmod{3}$$
$$x \equiv 2 \pmod{5}$$

Chinese Remainder Theorem

Consider the following congruences such that $n = n_1 n_2 \cdots n_k$ where $a_i, n_i \in \mathbb{Z}$ and $(n_i, n_j) = 1$.

$$x \equiv a_1 \pmod{n_1}$$
$$x \equiv a_2 \pmod{n_2}$$
$$\vdots$$
$$x \equiv a_k \pmod{n_k}$$

Then, there exists a unique solution to $x \equiv a \pmod{n}$.

Solve for x modulo 30 that satisfies the following congruences:

$$x \equiv 3 \pmod{2}$$
$$x \equiv 5 \pmod{3}$$
$$x \equiv 2 \pmod{5}$$

$(2,3)=1, \ (3,5)=1, \ (5,2)=1$

$2 \cdot 3 \cdot 5 = 30$

$\exists$ a unique solution to $x \equiv a \pmod{30}$

$$x \equiv 1 \pmod{2}$$
$$x \equiv 2 \pmod{3}$$
$$x \equiv 2 \pmod{5}$$

$x = 2k+1 \quad (k \in \mathbb{Z})$

$2k+1 \equiv 2 \pmod{3}$

$2k \equiv 1 \pmod{3}$

$x = 30x' + 17 \quad (x' \in \mathbb{Z})$

$30x' \equiv 0 \pmod{2}$

$\equiv 0 \pmod{3}$

$\equiv 0 \pmod{5}$

$k \equiv 2 \pmod 3$ $\quad$ ($\because (2,3)=1$)

$\therefore k = 3k' + 2$

$x = 2(3k'+2)+1$
$\quad = 6k'+5$

$6k'+5 \equiv 2 \pmod 5$

$6k' \equiv -3 \pmod 5$
$k' \equiv 2 \pmod 5$ $\quad$ ($\because (6,5)=1$)

$\therefore k' = 5k''+2$
$k = 15k''+8$
$x = 30k''+17$

$$\boxed{x \equiv 17 \pmod{30}}$$