Enoch Yu

Desargues' Theorem

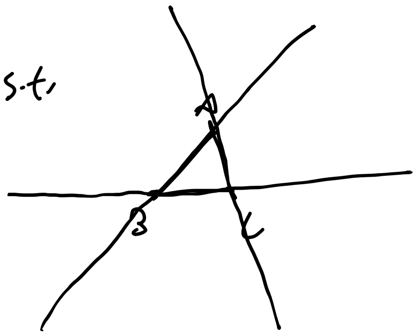
Two triangles ABC and A'B'C' are axially perspective if and only if they are centerally perspective.

↳ If the 3 lines that connects two corresponding points concur

↳ If the intersection of extended side lengths are collinear

Menelaus' Theorem

let X, Y, Z be points s.t.



$X \in AB$

$Y \in BC$

$Z \in CA$

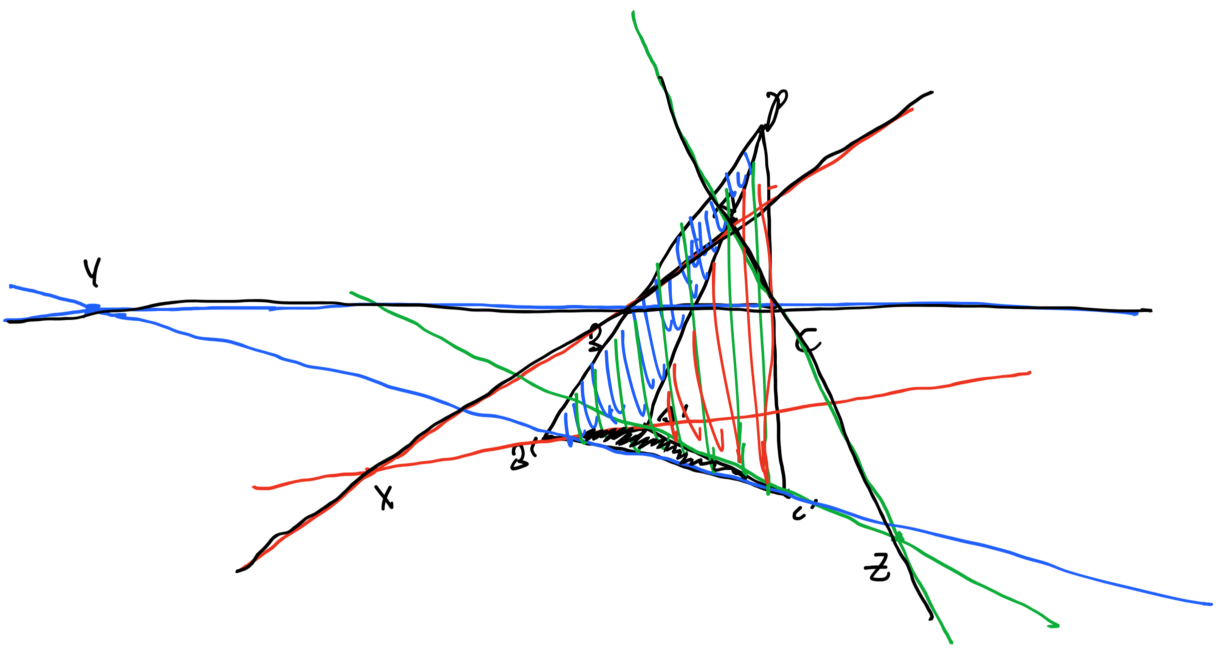
} X, Y, Z are collinear

cross-ratio,

$$\frac{XA}{XB} \cdot \frac{YB}{YC} \cdot \frac{ZC}{ZA} = -1$$

PF)

1. $\triangle ABC$ and $\triangle A'B'C'$ are axially perspective if they are centrally perspective.



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i) $\triangle PA'B'$, XB

$$\frac{XA'}{XB'} \cdot \frac{BB'}{BP} \cdot \frac{AP}{AA'} = 1$$

ii) $\triangle PB'C'$, YB

$$\frac{YB'}{YC'} \cdot \frac{BP}{BB'} \cdot \frac{CC'}{CP} = 1$$

iii) $\triangle PC'A'$, ZC

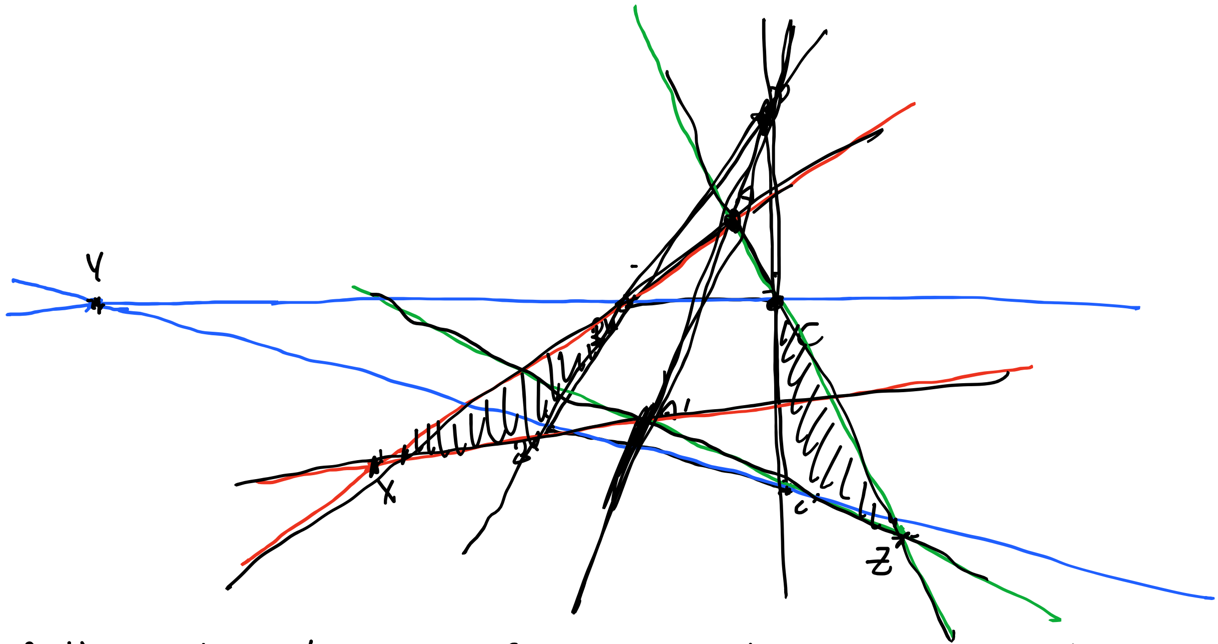
$$\frac{ZC'}{ZA'} \cdot \frac{AA'}{AP} \cdot \frac{CP}{CC'} = 1$$

$$\therefore \frac{XA'}{XB'} \cdot \frac{YB'}{YC'} \cdot \frac{ZC'}{ZA'} = 1$$

By converse of Menelaus' Theorem, X, Y, Z are collinear.

Thus, $\triangle ABC$ and $\triangle A'B'C'$ are axially perspective if they are centrally perspective.

2. $\triangle ABC$ and $\triangle A'B'C'$ are centrally perspective if they are axially perspective.



$BB'X$ and $CC'Z$ are in perspective from point Y.

$\therefore$ A, A', P are collinear and $P = AA' \cap BB' \cap CC'$

$\square$