Enoch Yu

Compute $\cos 252°$.

Solution I: Traditional Method

$\cos 252° = \cos 108° = -\sin 18° = \boxed{\dfrac{1-\sqrt{5}}{4}}$

let $x = 18°$ ①

$5x = 90°$ ②

$\sin 2x = \cos 3x$

$2\sin x \cos x = 4\cos^3 x - 3\cos x$

$2\sin x = 4\cos^2 x - 3$

$2\sin x - 4(1-\sin^2 x) + 3 = 0$ $\qquad \sin x > 0$

let $X = \sin x$

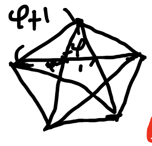
$4X^2 + 2X - 1 = 0$

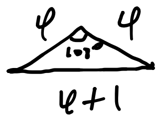
$X = \dfrac{-1 \pm \sqrt{5}}{4}$

$\therefore X = \dfrac{-1+\sqrt{5}}{4}$

Solution II: The Smart Way

$\cos 252° = \cos 108°$ ①





②

$\varphi = \dfrac{1+\sqrt{5}}{2}$

$\varphi^2 = \dfrac{3+\sqrt{5}}{2}$

$$\cos 108° = \frac{\varphi^2+\varphi^2-(\varphi+1)^2}{2\cdot\varphi\cdot\varphi}$$

$$= \frac{\varphi^2 - 2\varphi - 1}{2\varphi^2}$$

$$= \frac{\frac{3+\sqrt{5}}{2} - 1 - \sqrt{5} - 1}{3+\sqrt{5}} = \frac{3+\sqrt{5}-2\sqrt{5}-4}{2(3+\sqrt{5})} = \frac{(-1-\sqrt{5})(3-\sqrt{5})}{8}$$

$$= \frac{2-2\sqrt{5}}{8}$$

$$= \boxed{\frac{1-\sqrt{5}}{4}}$$