Enoch Yu

Prove that for all positive reals a, b, c, d,

$$\frac{1}{a} + \frac{1}{b} + \frac{4}{c} + \frac{16}{d} \geq \frac{64}{a+b+c+d}$$

PF) $(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{4}{c}+\frac{16}{d}\right) \geq 64$ 1

$$\underline{\left(\sqrt{a}^2+\sqrt{b}^2+\sqrt{c}^2+\sqrt{d}^2\right)\left(\left(\tfrac{1}{\sqrt{a}}\right)^2+\left(\tfrac{1}{\sqrt{b}}\right)^2+\left(\tfrac{2}{\sqrt{c}}\right)^2+\left(\tfrac{4}{\sqrt{d}}\right)^2\right)} \geq 64$$

$$\hookrightarrow \quad \geq (1+1+2+4)^2 = 64$$

$\square$