Enoch Yu

Given $z = \cos 40^\circ + i \sin 40^\circ$, express $\left|z + 2z^2 + 3z^3 + \cdots + 9z^9\right|^{-1}$ in the form of $a \sin b$.

$40^\circ = \frac{40}{180} \cdot \pi = \frac{2\pi}{9}$

$z = \cos\frac{2\pi}{9} + i\sin\frac{2\pi}{9}$

$= e^{i\frac{2\pi}{9}}$

$z^9 = 1$ $\quad z, z^2, \cdots, z^9$

$x^9 = 1$ $\quad \to 0$

2

$z^9 - 1 = 0$

$(z-1)(z^8 + z^7 + \cdots + 1) = 0$

$\to \sum_{k=1}^{9} k z^k$

1

$\to$ $S = z + 2z^2 + \cdots + 9z^9$

$-$ $Sz = z^2 + 2z^3 + \cdots + 9z^{10}$

$S(1-z) = z + z^2 + \cdots + z^9 - 9z$

$\therefore S = \frac{(z + z^2 + \cdots + z^9) - 9z}{1-z}$

$= \frac{-9z}{1-z}$

$\left|\frac{-9z}{1-z}\right|^{-1} = \left|\frac{1-z}{9z}\right|$

$= \frac{|1-z|}{|9z|}$

$= \frac{2\sin\frac{\pi}{9}}{9}$

$= \frac{2}{9}\sin 20^\circ$

2

$|z| = 1$

$|1-z| = \sqrt{(1-\cos\frac{2\pi}{9})^2 + \sin^2\frac{2\pi}{9}}$

$= \sqrt{2 - 2\cos\frac{2\pi}{9}}$

3

$= 2\sqrt{\frac{1-\cos\frac{2\pi}{9}}{2}}$

$= 2\sin\frac{\pi}{9}$