Enoch Yu

Let p and q be positive integers such that

$$\frac{5}{9} < \frac{p}{q} < \frac{4}{7}$$

and q is as small as possible. Find $q - p$.

$$\boxed{\frac{a}{b} < 1, \quad \frac{a}{b} < \frac{a+1}{b+1} < \cdots}$$

$$\boxed{\lim_{n \to b} \frac{a+n}{b+n} = 1}$$

$$\frac{35}{63} < \frac{p}{q} < \frac{36}{63}$$

$$\frac{35}{63} < \frac{36}{64} < \cdots < \frac{36}{63} \qquad < 1$$

$$\boxed{\frac{35+n}{63+n} = \frac{36}{63}}$$

$$63 \cdot 35 + 63n = 36 \cdot 63 + 36n$$

$$27n = 63$$

$$n = \frac{7}{3}$$

$$\frac{35+\frac{7}{3}}{63+\frac{7}{3}} = \frac{36}{63}$$

$$\frac{35+2}{63+2} <$$

$$\frac{35+1}{63+1} <$$

$$\frac{35}{63} <$$

$$\cancel{\frac{37}{65}} \quad / \qquad \frac{36}{64} = \frac{9}{16}$$

$$16 - 9 = 7$$

7