

$\forall a_i, b_i, \dots, z_i \in \mathbb{R}^+$

$$(a_1 + \dots + a_n)^{\lambda_a} (b_1 + \dots + b_n)^{\lambda_b} \dots (z_1 + \dots + z_n)^{\lambda_z} \geq a_1^{\lambda_a} b_1^{\lambda_b} \dots z_1^{\lambda_z} + \dots + a_n^{\lambda_a} b_n^{\lambda_b} \dots z_n^{\lambda_z}$$

for all $\lambda_i \geq 0$ s.t. $\lambda_a + \lambda_b + \dots + \lambda_z = 1$

$$(x_1^2 + \dots + x_n^2)(y_1^2 + \dots + y_n^2) \geq (x_1 y_1 + \dots + x_n y_n)^2$$

proof $\lambda_a = \lambda_b = \frac{1}{2}$

$$(a_1 + \dots + a_n)^{\frac{1}{2}} (b_1 + \dots + b_n)^{\frac{1}{2}} \geq a_1^{\frac{1}{2}} b_1^{\frac{1}{2}} + \dots + a_n^{\frac{1}{2}} b_n^{\frac{1}{2}}$$

$$x_i = \sqrt{a_i}, \quad y_i = \sqrt{b_i}$$

$$(x_1^2 + \dots + x_n^2)^{\frac{1}{2}} (y_1^2 + \dots + y_n^2)^{\frac{1}{2}} \geq x_1 y_1 + \dots + x_n y_n$$

$$(x_1^2 + \dots + x_n^2)(y_1^2 + \dots + y_n^2) \geq (x_1 y_1 + \dots + x_n y_n)^2$$

