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Let $a$, $b$, and $c$ be positive real numbers. Prove that

$$(a^5 - a^2 + 3)(b^5 - b^2 + 3)(c^5 - c^2 + 3) \geq (a+b+c)^3.$$

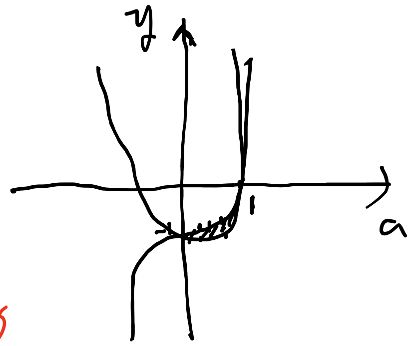
$a^5 - a^2 + 3 \geq$ ~~

$(\quad)(\quad)(\quad) \geq (a+b+c)^3$

$(a^3-1)(a^2-1) = a^5 - a^2 - a^3 + 1 \quad = a^5 - a^2 + 3$

$+a^3+2 \qquad +a^3+2$

$a^5 - a^2 + 3 \geq \boxed{(a^3-1)(a^2-1) \geq 0}$



$0<a<1, \quad a>1, \quad a=1$

$a^5 - a^3 - a^2 + 1 \geq 0$

$a^5 - a^2 + 3 \geq a^3 + 2$ ①

②

lemma $(a^3+2)(b^3+2)(c^3+2) \geq (a+b+c)^3$

proof $(a^3+2)^{\frac{1}{3}}(b^3+2)^{\frac{1}{3}}(c^3+2)^{\frac{1}{3}} \geq (a+b+c)^1$

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$

$\hookrightarrow$ Hölder's

$(a^3+1+1)^{\frac{1}{3}}(1+b^3+1)^{\frac{1}{3}}(1+1+c^3)^{\frac{1}{3}} \geq (a+b+c)^1$

$\hookrightarrow (a^3+1+1)^{\frac{1}{3}}(1+b^3+1)^{\frac{1}{3}}(1+1+c^3)^{\frac{1}{3}} \geq (a^3)^{\frac{1}{3}}1^{\frac{1}{3}}1^{\frac{1}{3}} + 1^{\frac{1}{3}}(b^3)^{\frac{1}{3}}1^{\frac{1}{3}} + 1^{\frac{1}{3}}1^{\frac{1}{3}}(c^3)^{\frac{1}{3}}$

$\geq a+b+c$

$\square$